Team Decision Theory and Information Structures

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Invited Paper

Abstract—This tutorial-survey paper introduces the problems of decentralized statistical decision making (team theory) where the decision makers have access to different information concerning the underlying uncertainties. Using a simple thematic example with variations, the paper introduces and explains various concepts and results of team theory as applied to economics, information theory, and decentralized control.

I. Introduction

CONSIDER the following simple everyday example of cooperative decision making. The example is somewhat contrived but is designed to emphasize the essentials of team theory by stripping away all unimportant details.[1] Mr. B who lives in Boston wishes to meet his partner Mr. H who lives in Hartford (about 100 miles away) in order to conduct some business in Worcester which happens to be located midway between Boston and Hartford. Their arrangement is that they will meet in Worcester the next day at noon if it doesn't rain. Because of various reasons, it became impossible for B and H to communicate further with each other after they have concluded their arrangement until their meeting. New England weather being what it is, an *uncertainty* has developed as to whether or not it is raining in Worcester when H and B are about to depart for their meeting. Of course each of them has access to his *own* local weather information in his city. This information is in turn correlated with the state of weather in Worcester as well as with the information received by the other person. Let us also assume that the nature of the business meeting is such that the presence of *both* partners is required. Thus a simple payoff matrix as illustrated in Fig. 1 may be postulated.

The question facing B and H is whether or not each should embark on the journey to Worcester. The problem is different from simple decision theory since each person must take into account in his decision what the other person may decide based on his own information. More specially, we assert that the main ingredients of a theory of team decisions are:

1) the presence of *different but correlated* information for each decision maker about some underlying *uncertainty*;
2) the need for *coordinated* actions on the part of all decision makers in order to realize the *payoff*.

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[1] Reader can easily translate this example to a more meaningful military context.

Rain in Worcester

Mr. B \ Mr. H	go	don't go
go	−4	−2
don't go	−2	5

Shine in Worcester

Mr. B \ Mr. H	go	don't go
go	10	−3
don't go	−3	0

Fig. 1.

If one or more of these ingredients are missing, then the problem simplifies, decouples, or trivializes. For example, if the cities involved were Boston, Tokyo, and London, then we do not expect that the local weather conditions will be correlated. In this case, it is intuitively clear that decision making will only depend on the prior expected weather conditions in London which we assume is known. The problem simplifies. It is also obvious that if the tasks to be performed by B and H do not require coordination and can be performed separately, then each person can make decisions independent of the other.

It should also be emphasized that we permit any kind of communication and agreement among the decision makers *beforehand*. Thus they can agree on taking any coordinated actions as a function of the information they separately have. For example, B can agree with H that he will go to Worcester only if it shines in Boston. However, H cannot be sure about the state of weather in Boston based on that of Hartford. This is to be contrasted with a two person optimization problem under conflict or noncooperative behavior where no pregame agreement can be enforced.

To proceed further with the solution of this example problem we need to specify some additional data, namely the correlations among weather conditions in the three cities. This we express as a joint probability distribution of three random variables ξ_B, ξ_H, ξ_W each can take on two values (rain or shine) as shown in Fig. 2.

Given the data in Figs. 1 and 2, we are then in a position to evaluate the expected payoff for any decision rule that may be adopted by B and H. Since the information state of each person is binary (rain or shine in his own city) and so are his choices, there are a total of four possible decision rules per person or sixteen decision rule pairs which we express as $u_B = \gamma_B(\xi_B)$, $u_H = \gamma_H(\xi_H)$. Note that the decision u_B can only depend on the information ξ_B and u_H on ξ_H. The payoff is a function L of u_B, u_H, and ξ_W. Thus the expected payoff for any γ_B and γ_H is

$$\bar{J} = \sum_{\xi} L(u_B, u_H, \xi_W) \Pr(\xi_B, \xi_H, \xi_W) \qquad \text{(I-1)}$$

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ξ_B	r	r	r	r	s	s	s	s
ξ_H	r	r	s	s	r	r	s	s
ξ_W	r	s	r	s	r	s	r	s
Prob.	0.25	0.05	0.1	0.1	0.1	0.1	0.05	0.25

Fig. 2

where

$$u_B = \gamma_B(\xi_B) \quad u_H = \gamma_H(\xi_H).$$

Note that the summation or expectation is meaningful *only when* the decision rules γ_B and γ_H are specified. The readers are invited to guess what the optimal decision rules for B and H should be (the answer with the optimal payoff is given at the end of the paper) and to go through the calculation in (I-1) for at least one decision rule pair.

II. A Formal Model and a Canonical Example for Team Decisions

We can formalize the example discussed in the previous section to develop a general model for team-theoretic decision problems. There are five principal ingredients:[2]

1) A vector of random variables $\xi = [\xi_1, \cdots, \xi_m]$ with given distribution $p(\xi)$. The random vector represents *all* the uncertainties that have bearing on the problem. They may be measurement noise, random disturbance, uncertain initial conditions, etc. ξ is often denoted as the "*states of nature*" or "*nature's decision.*"

2) A set of observations $z = [z_1, \cdots, z_n]$ which are given functions of ξ, i.e.,

$$z_i = \eta_i(\xi_1, \cdots, \xi_m), \quad i = 1, \cdots, n \tag{II-1}$$

In general z_i is a vector, and is known as the *information* or observation available to the ith decision maker (DM); The set $\{\eta_i | i = 1, \cdots, n\}$ is denoted as the *information structure* of the problem.

3) A set of decision variables $[u_1, \cdots, u_n] \equiv u$, one per DM. There is no loss of generality to assume u_i is scalar; vector u_i can always be decomposed into more DM's who happen to have access to the same observation z_i. The variables ξ, z, u are all assumed to take values in appropriate spaces Ξ, Z, U.

4) The *strategy* (decision rule, control law) of the ith decision maker is a map γ_i: $Z_i \rightarrow U_i$ which is simply a contingency plan of which decision to take under what circumstances (observations). We write

$$u_i = \gamma_i(z_i). \tag{II-2}$$

γ_i $\forall i$ is to be chosen from some admissible class of functions Γ_i.[3]

5) The loss (payoff) criterion of the problem is a map L: $\Xi \times U \rightarrow R$, i.e.,

$$\text{Loss} = L(u_1, \cdots, u_n, \xi_1, \cdots, \xi_m). \tag{II-3}$$

Note that for a given set of strategies tuples γ_i, $i = 1, \cdots, n$, L is a well-defined function of the ξ's, i.e.,

$$L(u, \xi) = L(\cdots, u_i = \gamma_i(\eta_i(\xi)), \cdots, \xi_j, \cdots)$$

Thus the expection of L with respect to $p(\xi)$ is well defined. We are now in a position to state the team decision problem as

$$\left\{ \begin{array}{l} \text{Find } \gamma_i \in \Gamma_i, \quad \forall i \text{ in order to.} \\ \text{minimize } J = E_\xi[L(u = \gamma(\eta(\xi)), \xi)] \\ \text{or} \\ \qquad \underset{\gamma \in \Gamma}{\text{Min}}\ J(\gamma) \end{array} \right\} \tag{II-4}$$

Equation (II-4) is known as the ***strategic*** form of the team problem. Note that conceptually (II-4) is a *deterministic* optimization problem albeit a very difficult one in general. It is a function (versus parameter) optimization problem and J is a functional. Even assuming all kinds of differentiability on the functions involved, we still face a calculus of variation problem in multiple independent variables (the ξ's). Furthermore, usually the space Γ has very little structure. Any reader who has attempted to solve the example problem in the previous section has experienced a taste of computational labor involved. Thus to go beyond brute force solution of (II-4) we need to either relax our optimization requirement or to impose more structure; both approaches will be pursued below.

Let us look at the team problem from the ith DM's viewpoint. Let $\bar{\gamma}_i$ denote the strategy of all other DM's. Assume $\bar{\gamma}_i$ is fixed and DM_i knows this.[4]

Then the problem facing DM_i is

$$\underset{\gamma_i \in \Gamma_i}{\text{Min}}\ J(\gamma_i, \bar{\gamma}_i) = \underset{\gamma_i \in \Gamma_i}{\text{Min}}\ E_\xi[L(u_i = \gamma_i(\eta_i(\xi)), \bar{\gamma}_i, \xi)]. \tag{II-5}$$

Since η_i's are fixed, z_i is a well-defined random variable, we can replace E_ξ by $E_{z_i}E_{\xi/z_i}$ where E_{ξ/z_i} stands for the expectation conditional on z_i. Thus

$$\begin{aligned} \underset{\gamma_i \in \Gamma_i}{\text{Min}}\ J(\gamma_i, \bar{\gamma}_i) &= \underset{\gamma_i \in \Gamma_i}{\text{Min}}\ E_{z_i}E_{\xi/z_i}(L(\gamma_i, \bar{\gamma}_i, \xi)) \\ &= E_{z_i} \underset{u_i \in U_i}{\text{Min}}\ E_{\xi/z_i}(L(u_i, \bar{\gamma}_i, \xi)) \end{aligned}$$

where the second equality comes from the fact that determining the optimal u_i for each z_i is the same as choosing γ_i. Consequently, we have the equivalent so-called *extensive form* of

[2]Readers are urged to use the example in Section I to help fix ideas in the general specification below.

[3]We do not consider mixed strategy since nothing is lost in the team case unless other constraints are present on Γ [27].

[4]Since this is a fully cooperative problem we can expect all DM to communicate with and inform each other during the design or solution phase of the problem.

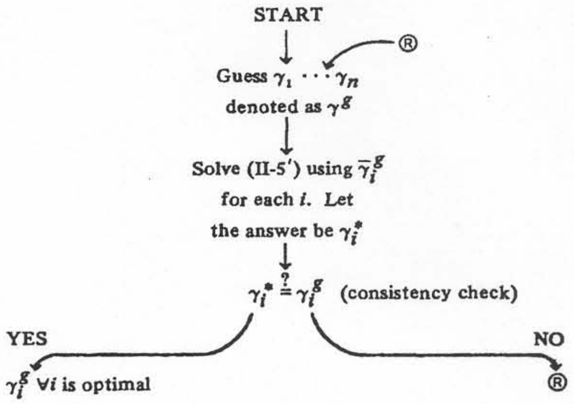

Fig. 3.

equation (II-5)

$$\underset{u_i \in U_i}{\text{Min}}\ E_{\xi/z_i}[L(u_i, \bar{\gamma}_i, \xi)] \equiv \underset{u_i \in U_i}{\text{Min}}\ J_i(u_i, z_i; \bar{\gamma}_i), \qquad \forall i. \tag{II-5'}$$

Note (II-5') is a parameter optimization problem for each z_i and fixed $\bar{\gamma}_i$, and is known as the *person-by-person optimality* requirement of the problem. Equation (II-5') is necessary for (II-4) but not vice versa. Only with additional assumption on L and/or other ingredients of the problem before (II-5') can guarantee (II-4). Note also, we have not completely eliminated the difficulties of the strategic form since (II-5') is still parametrized by $\bar{\gamma}_i$. Thus, to solve (II-5') requires in principle an iterative loop as shown in Fig. 3. This is in contrast with a one-person decision problem in the extensive form where no dependence on any part of γ is present in (II-5'). For this reason, equation (II-5') is often referred to as the *semistrategic form* of the problem.

Having developed the above general model, one might ask what type of issues should the model address. Two major questions and their derivatives have received most of the attention of the research effort in this area. Firstly, of course is the question "what should one do?" or "what is the optimal decision?" This requires the development of solution methodology associated with (II-4) or (II-5'). Secondly, we can raise the question "who should know what?" i.e., the design of the information structure η. In some sense, this is a more important and more difficult question since it presupposes the ability to answer the first question in some fashion and then follow it with a second stage optimization on η. However, the resolution of this second question will shed light on the problems of the value of information and the organizational form as informational efficient mechanisms of decentralized decision making.

To illustrate the solution ideas and issues discussed above and to develop further more specific results we shall restrict our attention to a particular class of problems. In fact, we shall discuss this class of problems in terms of a particular thematic example which with minor but interesting variations we shall employ for the rest of this paper. The understanding is that while results are obtained for this particular example, they are true for the entire class of problems; and the concepts developed have even more general validity. This claim will be made obvious as the paper develops.

Consider the following two person team problem. The decision variables are u_1 and u_2 which must be chosen to minimize the loss function;

$$L = \tfrac{1}{2}(x + au_1 + u_2)^2 + hu_1^2 + gu_2^2, \qquad a, g \geqslant 0, \qquad h > 0. \tag{II-6}$$

There are three random variables x, v_1, and v_2 defined for the problem with independent Gaussian distribution $x \sim N(0, \sigma^2)$, $v_1 \sim N(0, 1)$, and $v_2 \sim N(0, 1)$, respectively. Two observations

$$\begin{cases} y_1 = x + bv_1, & b > 0 \\ y_2 = x + cu_1 + dv_2, & c \geqslant 0,\ d > 0 \end{cases} \tag{II-7}$$

are possible on the system. The alert reader may have already noticed that y_2 is not in the form we had specified for a general team problem in Section I unless $c = 0$. We have introduced it in anticipation of further development with the example in the next two sections. The purpose is to illustrate further and different aspects of team problems by appropriate choice of these unspecified constants a, b, c, d, k, and g. In particular, nonzero value of c will introduce an entirely new dimension to the team problems discussed so far. In any case, to complete the specification for the first problem we stipulate

$$\text{(P1)} \begin{cases} a = b = d = 1, & c = 0 \\ h = g = \tfrac{1}{2} & \\ z_1 = y_1, & z_2 = y_2. \end{cases}$$

One interpretation of the problem is that there is a random initial state x; u_1 acts based on noisy observation $z_1 = x + v_1$ to bring the state to $x + u_1$; u_2 now acts based on $z_2 = x + v_2$ resulting in the final state $(x + u_1 + u_2)$; the object is to minimize the final state and the energy required to achieve it. Following Fig. 3, we guess

$$u_1 = k_1 z_1 \qquad u_2 = k_2 z_2 \tag{II-8}$$

Specializing (II-5') for (II-8) yields for $i = 1, 2$

$$\begin{cases} u_1 = -\tfrac{1}{2} E_{\xi/z_1}(k_2 z_2 + x) \\ u_2 = -\tfrac{1}{2} E_{\xi/z_2}(k_1 z_1 + x). \end{cases} \tag{II-9}$$

Using the standard formula for conditional expectation of Gaussian random variables and requiring (II-8) and (II-9) to hold for all z_1 and z_2 yields the consistency condition

$$\begin{bmatrix} 1 & \sigma^2/2(\sigma^2+1) \\ \sigma^2/2(\sigma^2+1) & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = -\begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{\sigma^2}{2(\sigma^2+1)} \tag{II-10}$$

which is a set of *linear* equations with the intuitively reasonable solution $k_1^* = k_2^* = -\sigma^2/(3\sigma^2 + 2)$. Is this solution optimal? (i.e., does it satisfy (II-4) in addition to (II-5')).

To resolve this let us denote the person-by-person optimal solution we just obtained as $u_i^* = \gamma_i^*(z_i) = k_i^* z_i$, $i = 1, 2$; and any other strategy $u_i = \gamma_i(z_i)$. Then by strict convexity of L in u_1 and u_2 we have

$$L(u_1, u_2, \xi) > L(u_1^*, u_2^*, \xi) + \left.\frac{\partial L}{\partial u_1}\right|_{u_1^*, u_2^*} (u_1 - u_1^*) + \left.\frac{\partial L}{\partial u_2}\right|_{(u_2 - u_2^*)} (u_2 - u_2^*). \tag{II-11}$$

Now taking expectation on both sides and substituting for u_i, u_i^* by γ_i and γ_i^*, respectively,[5] equation (II-11) becomes

$$J(\gamma_1, \gamma_2) \equiv E[L(u_1 = \gamma_1(z_1), u_2 = \gamma_2(z_2), \xi)]$$

$$> J(\gamma_1^*, \gamma_2^*) + E\left\{(\gamma_1 - \gamma_1^*) \left.\frac{\partial L}{\partial u_1}\right|_{u_1^*, u_2^*}\right\}$$

$$+ E\left\{(\gamma_2 - \gamma_2^*) \left.\frac{\partial L}{\partial u_2}\right|_{u_1^* u_2^*}\right\}$$

$$= J(\gamma_1^*, \gamma_2^*) + \left[E_{z_1}(\gamma_1 - \gamma_1^*) E_{\xi/z_1}\left\{\left.\frac{\partial L}{\partial u_1}\right|_{u_1^*, u_2^*}\right\}\right]$$

$$+ E_{z_2}\left[(\gamma_2 - \gamma_2^*) E_{\xi/z_2}\left\{\left.\frac{\partial L}{\partial u_2}\right|_{u_1^*, u_2^*}\right\}\right]$$

$$= J(\gamma_1^*, \gamma_2^*)$$

where the last equality is justified by (II-5′) which says that

$$\frac{\partial}{\partial u_i} E_{\xi/z_i}(L) = E_{\xi/z_i} \left.\frac{\partial L}{\partial u_i}\right|_{u_1^*, u_2^*} = 0.$$

The interchanging of expectation and partial differentiation is permitted since u_i is z_i-measurable. We have thus illustrated the first general result in linear-quadratic Gaussian (LQG) team theory [1].

Proposition 1: In LQG teams with $Q > 0$,

$$L = \tfrac{1}{2}u^T Q u + u^T S \xi, \qquad Q > 0, \quad \xi \sim N(0, \Sigma)$$

and

$$y = H\xi.$$

The unique optimal solution is linear in the information and is obtained by solving the person-by-person optimality conditions (II-5′).

III. Variations on a Theme

For the first variation on the thematic example discussed in the last section, let us consider the following information structure for u_1 and u_2 and specification of the parameters namely

$$(\text{P2})\left\{\begin{array}{l} z_1 = y_1 \equiv x + v_1 \\ z_2 = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \equiv \begin{bmatrix} x + v_1 \\ x + u_1 + v_2 \end{bmatrix} \\ a = b = d = 1, c = 1, h = g = \frac{1}{2} \end{array}\right\}.$$

The only difference of (P2) with (P1) lies in what u_2 knows, i.e., u_2 knows what u_1 knows and $c = 1$. As we mentioned earlier z_2 is not in the form of the general information structure we specified in Section II. However, consider

$$z_2' \equiv \begin{bmatrix} y_1 \\ y_2' \end{bmatrix} \equiv \begin{bmatrix} y_1 \\ y_2 - \gamma_1(z_1) \end{bmatrix} = \begin{bmatrix} y_1 \\ x + v_2 \end{bmatrix}. \qquad \text{(III-1)}$$

Knowing z_2 and γ_1 (re: footnote 4), we can generate z_2' and similarly for z_2 from z_2' and γ_1. Consequently any $u_2 = \gamma_2(z_2)$ can be realized by $u_2 = \gamma_2'(z_2')$ and vice versa. In other words, the information structure z_2' is equivalent to z_2. Thus (P2) with the equivalent z_2' comes under the class of LQG problems covered in Proposition 1, and (P2) can be solved by exactly the same method with the optimal strategies linear in the information

$$u_1 = k_1 z_1 \qquad u_2 = k_2^T z_2 \qquad \text{(III-2)}$$

where

$$k_1 = \frac{-\sigma^2}{3(\sigma^2 + 1)} \qquad k_2 = \begin{bmatrix} -\sigma^2/3(1 + 2\sigma^2) \\ -\sigma^2/2(1 + 2\sigma^2) \end{bmatrix}. \qquad \text{(III-3)}$$

The interpretation of (P2) is the same as (P1) except that y_2 (with $c = 1$) is to be considered as a noisy observation on the state of the system *after* u_1 has acted. In other words, we consider a dynamic system governed by $x_{t+1} = x_t + u_t$ evolving in time with initial state x. In this sense, the information structure z_2 has *perfect recall* since later DM always know what earlier DM knows. Thus if we regard u_1 and u_2 as decisions made by the same person at different times, we have just illustrated the well-known result.

Proposition 2: In one person LQG stochastic control problems with perfect recall, the optimal control law is linear.

It should be remarked that the linearity of the optimal control under this setup is independent of any assumption on $p(\xi)$ except Gaussianness. Since ξ represents all the randomness in the problem we can have *any* correlation among initial condition, measurement noise, and disturbances. Only when we wish to further express the solution in terms of Kalman filter plus deterministic controller (i.e., separation results) do we have to make additional assumptions on the form of $p(\xi)$, e.g., independence among the components of ξ.

More generally, let us consider

$$y_i = H_i \xi + D_i u \qquad \text{(III-4)}$$

where D_i must satisfy the causality conditions

$$\text{If } u_i \text{ acts before } u_j \text{ then } [D_i]_{j\text{th comp}} = 0 \qquad \text{(III-5)}$$

and the information structure

$$z_i = \begin{bmatrix} z_k & k \in E_i \\ y_i & \end{bmatrix}$$

where

$$E_i = \{k \mid D_{ik} \neq 0\}.$$

The information structure (III-5) says that if u_k affects the observation y_i (i.e., $D_{ik} \neq 0$) then u_i knows what u_k knows. This structure is a generalization of perfect recall and defines a partial order of inclusion on what various DM know. We denote this as a partial-nested (PN) information structure.

It should be clear from the development of (P2) that partial-nesting is sufficient to enable us to generate an equivalent information structure z_i' of the form specified in (III-1). Consequently we have Proposition 3.

Proposition 3: In a LQG team problem with PN information structure, the optimal strategies are linear and can be solved by solving a system of linear equations (generalization of (II-10)).

This is about as far as one can push the basic team results in Proposition 1 [11]. In fact if the information structure is linear then partial-nesting is necessary as well [29].

[5] Recall again that u_1 and u_2 are well-defined random variables *only* when the strategies γ_1 and γ_2 are specified.

For the second variation on our example, let us remove the assumption of perfect recall (or partial nestedness), i.e., we let

$$\text{(P3)}\begin{cases} a = c = d = h = 1, \quad b = g = 0 \\ z_2 = y_2 = x + u_1 + v_2. \end{cases}$$

The crucial difference is the fact that z_2 does not include y_1 in (P3). On the other hand "$b = g = 0$" is added only to aid our intuition. The interpretation of (P3) is that DM_1 knows x perfectly ($b = 0$) but it cost him energy ($h \neq 0$) to act. DM_2 has no cost ($g = 0$) on u_2 but his information about x is corrupted by noise and dependent on u_1. Since we do not have perfect recall, it is not possible to reduce this problem to that of (P2) or (P1). The "dynamics" or "order of action" definitely plays a part here. The type of information structure in (P3) is a special case of the more general form

$$z_i = \eta_i(\xi, u) \tag{II-1'}$$

subject to obvious causality requirements. Equation (II-1') is referred to as *dynamic* information structure and team problems with (II-1') are dynamic team problems as contrasted with (II-1) which are static. Note that this terminology does not depend on whether or not there is a dynamic system involved. Problem (P1) can certainly be interpreted from a dynamic system viewpoint as we have done. But it is still a static team problem. What makes the problem dynamic is the fact that the *information of latter DM depend on the decisions of earlier DM's.* In fact unless we specify the decision rule or strategy of the earlier DM, e.g., γ_1 in (P3), the information variable z_2 is not even a well-defined random variable. This difficulty is bypassed in (P2) when we derived an equivalent static information structure from the PN structure. This is not possible in (P3). More explicitly, note that $z_2 = x + \gamma_1(x) + v_2$ need not be a Gaussian random variable even though x and v_2 are unless γ_1 is linear in x. Thus computation of $E(x/z_2)$ may be a very difficult task since no easy formulas exist when z_2 has non-Gaussian distributions. Furthermore, looking at the payoff, we have $J = E_\xi[(x + \gamma_1(x) + \gamma_2(x + \gamma_1(x) + v_2))^2 + h\gamma_1^2(x)] \equiv J(\gamma_1, \gamma_2(\gamma_1))$. The thing to note here is that γ_1 enter J not only directly (from ku_1^2 term) but also indirectly through γ_2 (via z_2). Consequently, unless γ_2 is linear or some appropriate function J need not be convex in γ_1 even though L is convex (quadratic) in u_1. The point, which is not often appreciated even in open literature, is that we are dealing with an optimization problem in the $\gamma_1\gamma_2$ space. For the purpose of solution sometimes it is convenient to take the extensive form solution viewpoint, i.e., instead searching in γ space we try to determine the optimal u as function of z which is equivalent to constructing an optimal γ. But in order to ensure we have a nice optimization problem it is convexity and compactness *in the γ space* that is needed.

The computational as well as theoretical difficulties sketched above makes (P3) a most difficult problem. It is still unsolved today, some 12 years after Witsenhausen first analyzed it [2]. In fact, it may be surprising to the reader that despite all the work on optimization in the past, we do not even have a sufficient condition for optimality for (P3) similar to the usual second variation condition in function optimization. Optimization problem of the type $J(\gamma_1, \gamma_2(\gamma_1))$ involving composition of the optimizing functions simply have not been studied with any kind of systematic effort.

Although we have not been able to solve (P3) completely, we are not helpless. First of all, the conceptual solution by iterated guessing of γ in Fig. 3 can still be used. Following (P1) we guess $u_1 = k_1 z_1$ $u_2 = k_2 z_2$, we find that consistency and (II-5') requires that k_1 and k_2 satisfy a set of NONLINEAR (cf. (II-10)) algebraic equations.

$$k_1 = -\frac{(1 + k_2)}{2h + (1 + k_2)^2}$$
$$k_2 = -\frac{(1 + k_1)\sigma^2}{(1 + k_1)^2\sigma^2 + 1}. \tag{III-6}$$

Appropriate roots of (III-1) still constitute a person-by-person optimal solution for (P3). But unlike in (P1), they are NOT team optimal. Proof of this assertion was first pointed out in [2]. Consider the strategy $u_1 = \sigma \operatorname{sgn} x - x$. This has the effect of converting $x_1 = x + u_1$ to a random variable with a two point distribution at $\pm\sigma$. Since $z_2 = x_1 + v_2$, for large σ, z_2 is still essentially two point distributed. Then z_2 is a very good estimate of x_1 for large σ. Hence $u_2 = -\sigma \operatorname{sgn} z_2$ will almost surely cancel x_1 which implies $x_2 = x_1 + u_2 \approx 0$. Since there is no cost on u_2 it can be argued that for large σ the cost $J \cong kE(\gamma_1^2(x))$ can be made less than the person-by-person optimal cost realized by using γ_1 and γ_2 specified in (III-6). This is because $u_1 = \sigma \operatorname{sgn} x - x$ is on the average not necessarily a large number when σ is large.

On the other hand, a lower bound to the optimal cost can always be obtained for non-PN LQG dynamic team problem by "nestification" of the information structure, e.g., let $z_2 = [z_1, y_2]$ in (P3). Since adding information to any DM never hurts the optimal payoff,[6] Proposition 3 applied to the nestified version of (P3) will yield a lower bound to (P3). Thus if we denote J_L^* as the linear person-by-person optimal solution of (P3) in (6) and J_{PN}^* as the solution of the nestified version of (P3), we have the general.

Proposition 4: In non-PN dynamic LQG team problem the optimal J^* is bounded above and below by

$$J_{PN}^* \leq J^* \leq J_L^*. \tag{III-7}$$

The discussion about (P3) also illustrates another important conceptual point. The ability of u_1 to modify the information z_2 received by u_2 was used to advantage in deducing a superior nonlinear solution for (P3). We shall refer to this characteristic of dynamic information structure as "signaling" in the sense that DM_1 can control what DM_2 knows about the states of nature.

This signaling effect can be made more transparent if we consider

$$\text{(P3')} \quad \{\text{(P3) with } d = 0 \text{ and } z_2 = y_2 = u_1\}.$$

In this case we can take $u_1 = \epsilon x$ and $u_2 = -(1 + \epsilon) u_1/\epsilon$ and J^* can be made arbitrarily close to zero by taking ϵ small enough. Here the role of u_1 is purely to inform u_2. Because $d = 0$ and $y_2 = u_1$ are equivalent to infinite precision in the transmission of information, we can visualize u_1 as a double precision number. The most significant half of the digit positions of the number is used to optimize J directly (u_1 optimal is zero in this particular case); the least significant half can be used to

[6] Any strategy that can be realized before can still be realized with the enlarged information structure.

carry signal information (the value of x) without affecting the payoff substantially. This phenomenon of hiding information in control action has been referenced to as "transparency" of information [3].

In traditional control problems, we introduced the concept of "dual control" to summarize the idea that the purposes of control are two fold. One is to reduce error; the other to improve *our own* knowledge about any uncertainty. We can extend this concept to decentralized control problem and introduce the idea of "triple control."[7] That is, a third purpose of control is to reduce *other DM's* uncertainties via "signaling."

In general, these three purposes of control are all present in a given optimal strategy for a general team problem with the information structure of (II-1′). If all stochastic effects are absent, e.g., in deterministic problems, only the first purpose remains. In (P3) since each person only acts once, the second purpose is absent. But DM_1 does more than just signaling since u_1 also modifies the payoff function u_2 sees (u_2 observes a noisy x_1 and attempts to cancel x_1 not x). In order to isolate the phenomenon of signaling, we shall introduce a third variation of our thematic example.

IV. Signaling and Information Theory

Consider the problem

$$\text{(P4)} \quad \min_{\gamma_1,\gamma_2} E\{[x - \gamma_2(x + \gamma_1(x) + v_2)]^2 + h\gamma_1^2(x)\}.$$

This is (P3) with arbitrary h and $a = 0$. Since $\gamma_1(x)$ is arbitrary so is $x + \gamma_1(x)$ we consider the slightly modified version of (P4)

$$\text{(P4}'\text{)} \quad \min_{\gamma_1,\gamma_2} E[(x - \gamma_2(\gamma_1(x) + v_2))^2] \qquad \text{s.t. } E(\gamma_1^2(x)) \leq \alpha$$

where we have regarded h in (P4) as a LaGrange multiplier for the constraint $E(\gamma_1^2) \leq \alpha$. Problem (P4′) has a natural information-theoretic interpretation. DM_1 observes $x \sim N(0, \sigma^2)$ and encodes his observation as u_1 which is sent over an additive Gaussian channel ($z_2 = u_1 + v_2$, $v_2 \sim N(0, 1)$) subject to power constraints. DM_2 must decode z_2 to produce u_2 which serves as an estimate of x. The criterion is simply mean-square error. In (P4′), the sole purpose of u_1 is to signal to DM_2 about the state of nature x; its effect does not enter the criterion function directly. In this sense information theory can be viewed as the simplest kind of team decision theory with non-PN dynamic information structure in which only the "signaling" aspect is involved. To be sure, classical Shannon information theory, e.g., in the case of Gaussian source ($x \sim N(0, \sigma^2)$) and Gaussian channel ($z_2 = u_1 + v_2, v_2 \sim N(0, 1)$) address somewhat different issues. In the context of (P4′) it considers the following generalization of (P4′). Let $x \equiv [x_1, \cdots, x_n]$ where x_i are independent samples of x from $N(0, \sigma^2)$, DM_1 is allowed to send messages $u_1 \equiv [u_{11}, u_{12}, \cdots, u_{1m}]$ where each component $u_{1i} = \gamma_{1i}(x_1, \cdots, x_n]$ is allowed to depend on all x_j, $j = 1, \cdots, n$. The received signal $z_2 \equiv [z_{21}, \cdots, z_{2m}]$ is defined in the natural way $z_{2i} = u_{1i} + v_i$ where v_i are independent samples from $N(0, 1)$. DM_2 then decodes z_2 to produce $u_2 \equiv [u_{21}, \cdots, u_{2i} = \gamma_{2i}(z_{21}, \cdots, z_{2m}), \cdots, u_{2n}]$ as estimate of x. Thus we have the following Information Theory (IT) problem

$$\text{(P4}'\text{-IT)} \begin{cases} \min_{\gamma_1,\gamma_2} E\left[\frac{1}{n}\sum_{i=1}^{n} (x_i - \gamma_{2i}(z_2))^2\right] \\ \text{s.t. } z_{2i} = u_{1i} + v_i, \qquad i = 1, \cdots, m \\ \text{and } \frac{1}{m}\sum_{i=1}^{m} E(\gamma_{1i}^2) \leq \alpha. \end{cases}$$

In the limit as $n, m \to \infty$ but $m/n \equiv R$ = constant, we can visualize $(1/n)E[\sum_{i=1}^{n}(x_i - \gamma_{2i}(z_2))^2] \equiv D$ as the average distortion and $(1/m)\sum_{i=1}^{m} E(\gamma_{1i}^2) \equiv \alpha$ as the average cost.[8] Solution of (P4′-IT) in this asymptotic case then gives the minimum distortion D^* as a function of R. This is essentially what is known as the *rate-distortion function* in information theory. Intuitively, D^* varies roughly inversely as R. The nontrivial results in IT are the demonstration that this D^* can in fact be achieved and calculated without explicitly solving the optimal encoder and decoder pair (γ_1, γ_2). In (P4′-IT), we let each component of u_1 and u_2 depend on the entire x and z_2 vector, respectively. This is known as *block coding and decoding*. It means in the case that x_i's are sequentially generated that in order to send the first message u_{11}, one may have to *wait* initially until x_n is generated. A similar delay may be required in decoding if the channel accepts messages sequentially, i.e., generation of u_{21}, the estimate of x_1. In the context of team theory or decentralized control such delay may be unacceptable. Instead we may consider what might be called the Real Time Information Theory (RTIT) problem[9]

Problem (P4′-IT) with the added constraint

$$\text{(P4}'\text{-RTIT)} \begin{cases} u_{1i} = \gamma_{1i}(x_1, \cdots, x_i), & \forall i \\ u_{2i} = \gamma_{2i}(z_{21}, \cdots, z_{2i}), & \forall i. \end{cases}$$

Very little is known about (P4′-RTIT) type of problems. Indications are furthermore that they are very difficult problems [4]. However, result from (P4′-IT) can always serve as another lower bound to solutions for (P4′). In fact, we have

$$J^*\,(\text{P4}'\text{-IT}) \leq J^*\,(\text{P4}'\text{-RTIT}) \leq J^*\,(\text{P4}') \tag{IV-1}$$

since each case successively enlarges the space of admissible (γ_1, γ_2) pairs.

Various known results in connection with real time "signaling" inspired by knowledge of classical information theory can be found in [5].

V. Application to Economics

Team theory was originally developed by economists to model economic problems under the constraint of imperfect information. Some of the original motivations for the development of the theory have been largely eliminated through the wide availability of computer information networks, e.g., airline ticket reservation systems. However, this merely escalated the level at which decentralization consideration becomes important. Current interests in distributed data base system and command-control-communication problems are simple

[7] Terminology due to P. Varaiya.

[8] It should be intuitively clear that one always uses the maximal power allowed.

[9] For simplicity we have assumed $m = n$ here. If $m \neq n$ some obvious modification is necessary. The principle is that decisions u_{1i} or u_{2i} cannot depend on samples not yet generated or messages not yet received.

manifestations of the cost of acquisition and management of information and need for decentralization. We shall describe below some illustrative examples of the more recent team-theoretic considerations in economics.

Let us consider another variation of our thematic example with a resource allocation interpretation. Let the payoff be

$$L = -\{(\tfrac{1}{2}u_1^2 + \alpha_1 u_1) + (\tfrac{1}{2}u_2^2 + \alpha_2 u_2) + \tfrac{1}{2}(u_1 + u_2 - x)^2\} \quad \text{(V-1)}$$

where $\alpha_1 \alpha_2$, x are Gaussian-distributed r.v.'s with specified $p(\alpha_1, \alpha_2, x)$. If we choose $a = 1$, $h = g = \frac{1}{2}$ and maximize J instead of minimizing then (V-1) is essentially the same as that of (P1) save for the addition of the $\alpha_1 u_1, \alpha_2 u_2$ term. The interpretation of (V-1) is however quite different; $-(\frac{1}{2}u_i^2 + \alpha_i u_i)$, $i = 1, 2$ is the production function of DM_i who shares a common resources x with the other DM. The soft resource constraint is represented by the third term of J in (V-1) which might be a plausible substitute for the constraints $u_1, u_2 > 0$ and $u_1 + u_2 = x$. Thus the object is to maximize production subject to resource constraints.[10]

Let DM_i's information be

$$z_i = \alpha_i \quad \text{(V-2)}$$

which is information on local production parameters. Also for simplicity, we shall assume α_1, α_2, x are zero mean, unit variance, and independent. Neither of these assumptions are necessary. However, not assuming them complicates the algebra considerably and obscures the point we wish to make below. The zero mean assumption can be justified by viewing u_1, u_2 as perturbations from nominal production decisions and x as perturbation in resources. However, it should be pointed out that for independent α_1, α_2, and x, the team aspect of the problem at this point is trivialized. The problem decouples and the optimal solution is given by

$$u_1^* = -\alpha_1/2$$
$$u_2^* = -\alpha_2/2, \qquad J^* = 0. \quad \text{(V-3)}$$

Now suppose there is a coordinator to whom DM_1 and DM_2 report their individual observation z_1 and z_2 and who in turn broadcasts a message back to the DM's. For example let the message be

$$\lambda = -\frac{x + \alpha_1 + \alpha_2}{2} \quad \text{(V-4)}$$

then we can show directly that for

$$\begin{cases} u_1^{\#} = -\alpha_1 - \lambda & \text{(V-5)} \\ u_2^{\#} = -\alpha_2 - \lambda \\ J^{\#} = \frac{1}{4} > J^*. & \text{(V-6)} \end{cases}$$

The solution (V-5) and (V-6) has the following interpretation, i.e., they also solve the problem

$$\text{Max} -(\tfrac{1}{2}u_1^2 + \alpha_1 u_1) - (\tfrac{1}{2}u_2^2 + \alpha_2 u_2) \text{ s.t. } u_1 + u_2 = x \quad \text{(V-7)}$$

where everyone has complete information about α_1, α_2 and x. "λ" above plays the role of LaGrange multiplier or the "price" the coordinator charges for the use of the resource x; $u_i^{\#}$ is then the profit maximizing decision of "production revenue-cost of resources." Since there is more information for everyone under (V-7) then (V-1), it is not surprising that $J^{\#} > J^*$. What is interesting is that the solution does *not* require DM_i to know about x or the other production parameters α_j. The coordinator only needs to broadcast a single number, the price λ. This is the well known Arrow-Hurwicz algorithm for decentralized resource allocation [22]. The saving afforded by this algorithm grows as the number of DM increases. What is even more interesting from team theory point of view is that as the number of DM grows we have

$$J = \sum_{i=1}^{n} (-\tfrac{1}{2}u_i^2 - \alpha_i u_i) \text{ s.t. } \sum_{i=1}^{n} u_i = x \quad \text{(V-7')}$$

and

$$u_i^{\#} = -\alpha_i - \lambda, \qquad i = 1, \cdots, n \quad \text{(V-8)}$$

where

$$\lambda = -\frac{x}{n} + \frac{1}{n}\sum_{i=1}^{n} \alpha_i. \quad \text{(V-9)}$$

Since α_i's are independent samples from a zero-mean distribution, the variance of λ decreases with increasing n, i.e., λ becomes more and more predictable. Consequently, the percentage difference $(J^{\#} - J^*)/J^{\#}$ decreases to zero. Intuitively, the reason for this is the fact that as the number of team member grows without limit the sample distribution of different member types (represented by the different production parameters α_i) approaches the underlying distribution. In other words, every α_i value that could arise will in fact be present. Since each individual DM_i is coupled to the other DM's only through their aggregate demands on the shared resources, no more uncertainty is left in the problem. No communication among members is as good as complete exchange of information. It is only necessary for the coordinator to broadcast the value x at the beginning. No reporting by the members are necessary. The aggregate demand of all the DM's can be predicted beforehand.

Another interesting phenomenon associated with (V-1) is the question of incentive in teams [7]. We can interpret the first two terms in (V-1) as the production function of two divisions of a conglomerate company who are competing for a common resources (e.g., capital) available from the headquarters. Since the performance of (or reward to) each division is measured by its own production, i.e., $\frac{1}{2}u_i^2 + \alpha_i u_i$, there are incentives for each division to misrepresent their production parameter α_i in the hope that they may be allotted a larger share of the scarce resources x. For example, left to their own devices, each division would want $u_i = -\alpha_i$. On the other hand, the center according to (V-1) would want $u_1 = (x - 2\alpha_1 + \alpha_2)/3$, $u_2 = (x - 2\alpha_2 + \alpha_1)/3$. Thus it is tempting for each division to report a somewhat larger α_i, say $\hat{\alpha}_i$, to the center. Suppose it is not possible for the center to verify the truth of the reported $\hat{\alpha}_i$'s.[11] The question that then arises as to whether or not it is possible for the center to devise an incentive structure *based on only the knowledge it has* (which may contain misinformation) so as to induce each team member to report the

[10] This is a grossly simplified interpretation. For a more elaborate model and explanation see [6].

[11] This is consistent with the assumptions of the team problem. Otherwise, the center might as well solve the entire problem and each division is merely told what to do. We assume here that the members are free to report whatever they wish about α_i which now represents their decisions. The center then chooses u_1, u_2 based on these reported α_i's.

truth. This question has additional application in areas such as allocation of the cost of public goods and project which must be paid for by members who partially benefit from such projects. For details see [8] and references therein. The basic idea however can be easily captured via our example.

Let $\hat{\alpha}_1$, $\hat{\alpha}_2$ be the production parameter reported by the member divisions to the center. The center accepts these values as truthful and compute the optimal allocation

$$\hat{u}_1 = \frac{x - 2\hat{\alpha}_1 + \hat{\alpha}_2}{3} \qquad \hat{u}_2 = \frac{x - 2\hat{\alpha}_2 + \hat{\alpha}_2}{3} \tag{V-10}$$

on this assumption, and declare an incentive structure which says that the ith division will receive

$$J_i = -(\tfrac{1}{2}\hat{u}_i^2 + \alpha_i\hat{u}_i) + [-(\tfrac{1}{2}\hat{u}_j^2 + \hat{\alpha}_j\hat{u}_j) - \tfrac{1}{2}(x - \hat{u}_i - \hat{u}_j)^2] - A_i(\hat{\alpha}_j). \tag{V-11}$$

Equation (V-11) has the following meaning. The first term is the actual payoff earned by division i based on the allocation $\hat{u}_i$. The second term has the meaning as what the center "thinks" as the contribution of the rest of the company based on the report of the team members. The third term is a term dependent only on the parameter of the other team member. It is to be chosen so that

$$A_1(\hat{\alpha}_2) + A_2(\hat{\alpha}_1) > [-(\tfrac{1}{2}\hat{u}_1^2 + \hat{\alpha}_1\hat{u}_1) - \tfrac{1}{2}(x - \hat{u}_1 - \hat{u}_2)^2] + [-(\tfrac{1}{2}\hat{u}_2 + \hat{\alpha}_2\hat{u}_2) - \tfrac{1}{2}(x - \hat{u}_1 - \hat{u}_2)^2]. \tag{V-12}$$

In this sense, we can view the sum of the second and third term in equation (V-11) as a form of "profit sharing" payoff declared by the center payable to division i. Equation (V-12) is to insure that the center will always have enough money to pay each division's profit-sharing share. Now maximizing J_i with respect to $\hat{\alpha}_i$, it can be directly verified that the optimal $\hat{\alpha}_i$ is in fact equal to α_i, the true value. In other words, the incentive structure is such that any advantage that can be obtained from the first term in (V-12) by not telling the truth about α_i will be more then offset by the second and third term. In fact, the profit-sharing terms may be negative in extreme cases. To complete the story, we must demonstrate that it is always possible to find A_1 and A_2 to satisfy (V-12) and hence the postulated incentive structure. Consider

$$A_1 \equiv \underset{u_2,u_1}{\text{Max}} \left[-\frac{1}{2}u_2^2 - \hat{\alpha}_2 u_2 - \frac{\theta_2}{2}(x - u_1 - u_2)^2 \right]$$
$$A_2 \equiv \underset{u_2,u_1}{\text{Max}} \left[-\frac{1}{2}u_1^2 - \hat{\alpha}_1 u_1 - \frac{\theta_1}{2}(x - u_1 - u_2)^2 \right] \tag{V-13}$$

where

$$\theta_1 + \theta_2 = 2.$$

It is clear that $A_1 + A_2$ satisfies (V-12) since $A_1 + A_2$ is the largest value that could be taken on by the right-hand side of (V-12).

A third example application of team-theoretic ideas in economics is to market signaling by Spence [10]. This problem actually involves dynamic information structure and is further related to the information-theoretic issues discussed in Section III. In terms of our formulation, the problem can be stated as follows:

An employer must hire someone for a job without knowing how productive that individual will be. In other words, the employer has imperfect information about an individual's ability. Spence suggests that the employer can improve his information by looking on the job application for some signal, such as educational level. The employer offers wages based on the signal he sees; that is, a person with more education is offered higher wages, because the employer believes that the higher education indicates higher ability. The individual applying for the job, on the other hand, knowing he will receive wages based on his educational level, must decide how much education to get, taking into consideration that education is costly. Let DM_1 = all potential employees considered together, DM_2 = the employer, x = an individual's ability (known to that individual, but not to the employer), u_1 (or u_1 + noise) = educational level, and u_2 = wages. The payoff or loss function of DM_2 is $[x - u_2]^2$; he does not wish to overpay or underpay with respect to x. The payoff of DM_1 is simply the net profit $u_2 - c(u_1, x)$ where c is the cost of signaling. Thus we have

$$\text{(P5)} \begin{cases} \text{Find} & u_1 = \gamma_1(z_1) \quad u_2 = \gamma_2(z_2) \text{ to} \\ \text{Max} & J_1 = E[u_2 - c(u_1, x)] \\ \text{Min} & J_2 = E[(u_2 - x)^2] \\ \text{where} & z_1 = x \\ & z_2 = u_1. \end{cases}$$

There is a great deal of commonality between (P5) and (P4$'$) discussed earlier. (P4) is a true team theory problem. Both u_1 and u_2 are interested in minimizing $E(u_2 - x)^2$ subject to some cost or power constraints on u_1. Here, only u_2 is interested in minimizing $E(u_2 - x)^2$, while u_1 has his own payoff with cost considerations. Thus instead of a cooperative minimum they are seeking a person-by-person noncooperative optimum. The important point is that the information structure is dynamic ($z_2 = u_1$). Agent 1 can in fact use his action u_1 to signal agent 2 about the state of nature x. Many interesting economic implication and interpretation of such a model can be derived from this signaling structure. For further details see [9], [10].

VI. Decentralized Control as Team Decision Problems

We have so far in our discussion paid minimal emphasis on "dynamics." Although in some versions of the thematic example the notion of who acts first, i.e., order of decision, is important, the usual feature of dynamical systems in terms of ordinary or partial differential or difference equations is absent. This is deliberate for two reasons. Firstly, we wish to clarify the concepts involved without additional complexities. Secondly, from the viewpoint of team theory and information structures, "dynamics" in its usual sense is only a complicating and sometimes misleading detail. It adds little to the conceptual problem. We shall explain what we mean.

Consider a stochastic dynamic system described by an ordinary difference equation

$$x_{t+1} = f(x_t, u_t, w_t), \quad t = 1, \cdots, T \tag{VI-1}$$

where w_t is a given stochastic sequence with specified $p(w_1, \cdots, w_{T-1})$; u_t are the control variables and x_t, the state variables. Let the cost of controlling this system be measured by

$$J = E[\phi(x_T)]. \tag{VI-2}$$

We then have the beginning of an optimal stochastic control problem or sequential decision problem.

From team theory viewpoint, we can regard u_t as the choice of a distinct decision maker $\mathrm{DM}_t\ i = 1, \cdots, T$, versus the tth decision of a single controller as it is customary done in control theory. Now substituting (VI-1) repeatedly in (VI-2) we get

$$\begin{aligned} J &= E[\phi(f(x_{T-1}, u_{T-1}, w_{T-1}))] \\ &= E[\phi(f(f(x_{T-2}, u_{T-2}, w_{T-2}), u_{T-1}, w_{T-1}))] \\ &\vdots \\ &\triangleq E[L(u_1, u_2, \cdots, u_{T-1}, w_1, \cdots, w_{T-1}, x_1)]. \quad \text{(VI-3)} \end{aligned}$$

In the notation of Section II, $u_1, \cdots, u_{T-1}$ are the decision variables of the team and $w_1, \cdots, w_{T-1}, x_1$ are Nature's decisions, the random variable ξ. The point to emphasize is that the so-called "state variables" in dynamic system is of secondary importance here. Similarly, observations on the dynamic system such as

$$z_T = h_t(x_t, v_t) \quad \text{(VI-4)}$$

where v_t is another given stochastic sequence can be handled by substituting out x_t using (VI-1) to yield

$$\begin{aligned} z_t &\triangleq \eta_t(u_1, \cdots, u_{t-1}, w_1, \cdots, w_{t-1}, x_1, v_t) \\ &\triangleq \eta_t(u, \xi). \quad \text{(VI-5)} \end{aligned}$$

Equation (VI-5) is a generalization of the model of information structure we introduce in (II-1)–(II-7). It is here in the information structure that "dynamics" in the usual sense plays a crucial role in our problems. The presence of u in (VI-5) makes the information variable dependent on the decision variable, and hence the strategies, of the earlier decision makers. In our discussion earlier with respect to (P3), the difficulties introduced by (VI-5) were discussed in detail. Unless additional structure is assumed for (VI-5), relatively little can be done. On the other hand, aside from this basic difficulty, the presence of "dynamics" in a team problem poses no additional problems. For other examples on reducing dynamics to the team model of Section II see [11]. It is for this reason that *deterministic* dynamic control problem involving several players are often solvable since the dual and triple control aspects of the problem are absent.

It is appropriate at this point to discuss what appropriate structural assumption can we make regarding η in the dynamic case. In the one controller case, perfect memory is a standard as well as a reasonable assumption for η. This reduces the problem to a special case of the PN information structure. However, when there are actually several controllers involved each controlling one or more dynamic systems, group perfect memory (in the sense that every controller knows the information available to every other controllers in addition to perfect memory) simply assumes away the problem as well as not being acceptable in a practical sense. In this sense optimal decentralized stochastic control is an extremely difficult problem inasmuch as we cannot even solve the simplest problem (P3).

A very comprehensive discussion of the interplay between information structure and strategies in the setting of discrete time decentralized stochastic control can be found in [12]. One of the best known examples in this area is the so-called "one-step delay sharing information structure" [12], i.e., each controller shares all but current information with others. This is a special PN structure adapted to decentralized dynamic systems. Heuristically, since all past information are shared, the dependence of information on past strategy can be eliminated leaving only the part of current information which are independent from past strategies and which are different. The problem than reduces to the static case of (P1) and Proposition 2. Further extensions of ideas along this line can be found in [13], [14]. One particularly surprising result in delay sharing information structure is a counter example independently discovered [15], [16] which refutes a very intuitive conjecture stated in [12]. Briefly this is the situation. Let two controllers share information with n-step delay for $n > 1$ (say, $n = 2$). The information structures at time t are

$$\eta_t^1 = \{y_t^1, y_{t-1}^1, C_{t-2}\}$$
$$\eta_t^2 = \{y_t^2, y_{t-1}^2, C_{t-2}\}$$

where C_{t-2} is the common information both controllers share about the dynamic system two steps ago and y_t^i, y_{t-1}^i are the private information the ith controller has about the system. The conjecture being that there is no loss of generality if we restrict our consideration of control strategies to the class

$$u_t^i = \gamma_t^i(y_t^i, y_{t-1}^i, p(x_{t-1}/C_{t-2}))$$

where $p(x_{t-1}/C_{t-2})$ is the conditional distribution of the "state" of the dynamic system at $t - 1$ given C_{t-2}. The failure of this conjecture has to do with our erroneous attachment to and preoccupation with the concept of the "state" of a dynamic system which is a leftover from deterministic considerations. If one insists, then for the case of $n = 2$, the "state" should be the triple x_{t-1}, u_{t-1}^1, u_{t-1}^2 since only by knowing all three values can we eliminate the dependence of the private information y_t^i, y_{t-1}^i from past strategies. For more details see [26].

Since optimal solutions are difficult to obtain, one is often forced to ask a less ambitious question, such as what interesting properties of decentralized control system can we discover if we restrict *a priori* the class of admissible strategies?

In one sense, by restricting the admissible strategies to some narrowly defined class (e.g., linear and finite dimensional in information variables), one has assumed away the informational aspect of the problem. Even if optimum can be achieved within the class, there is usually no guarantee that performance cannot be significantly improved if one goes outside the class. Thus the *raison d'être* for this approach lies in other properties of the optimal solution. For example, the periodic coordination scheme of [17] suggests organizational imperatives which are intuitively reasonable. Also qualitative questions such as the controllability, observability, and stability of decentralized controlled dynamic system under certain class of strategies are often of interest. Since these matters lie outside the range of team theory defined in the orthodox sense of the term, we refer readers to a recent comprehensive survey on the subject [18].

There is one other point worth emphasizing in connection with decentralized control and information structure. In a team problem every decision maker is by definition interested in the same team payoff, i.e., he is fully cooperative. It is natural to assume that every DM know the strategy of every other DM. In this case knowing what DM_j knows is equivalent to knowing what he will do, i.e., $u_j = \gamma_j(\eta_j)$. Nothing is gained by having an information structure which includes the explicit knowledge of u_j if the information structure already contains η_j. However, in many person control problems, different players may not share the same payoff. In such cases, there is considerable difference between the information structure

$\eta_i = \{\cdots u_j, \eta_j, \cdots\}$ and $\eta_i' = \{\cdots, \eta_j, \cdots\}$. With the first structure, the possibility of directly influencing DM_j's decision by DM_i exist. Knowing both u_j and η_j, DM_i is in fact in a position to verify whether or not DM_j indeed employed a certain strategy γ_j. DM_i can threaten to undertake certain actions unless DM_j behaves in certain manner. For example, the strategy γ_i can be as follows

$$u_i = \cdots + f(u_j - \gamma_j^d(\eta_j)) + \cdots \triangleq \gamma_i(\eta_i) \qquad \text{(VI-6)}$$

where $\gamma_j^d(\eta_j)$ is the desired strategy for DM_j by DM_i and f is some penalty or incentive function used to induce the proper behavior of DM_j. Strategies of the form of (VI-6) has been used to

1) derive nonunique Nash equilibriums in nonzero sum control problems [19];
2) convert a Stackelberg control problem to a team control problem [20], [21].

The last point is sufficiently interesting to warrant a simple illustration of its main concept particularly since it appears more complicated than it really is in the general literature. Consider $J_1 = L_1(u_1, u_2)$ and $J_2 = L_2(u_1, u_2)$. Suppose further that u_2 can use a strategy which depends on u_1, i.e., $u_2 = \gamma_2(u_1)$. Then for given γ_2, u_1 must solve the problem of $\text{Max}_{u_1} L_1(u_1, \gamma_2(u_1))$. The result is in general given by

$$u_1 = \gamma_1^0(\gamma_2). \qquad \text{(VI-7)}$$

Knowing the response of u_1 in (7), u_2 faces the problem of $\text{Max}_{\gamma_2} L_2(u_1 = \gamma_1^0(\gamma_2), u_2 = \gamma_2(u_1))$. Suppose u_1^t and u_2^t are the team solution to $\text{Max}_{u_1} \text{Max}_{u_2} L_2(u_1, u_2)$. We can choose a $u_2 = \gamma_2^*(u_1)$ such that

$$u_2^t = \gamma_2^*(u_1^t) \qquad \text{(VI-8a)}$$

$$u_1^t = \gamma_1^0(\gamma_2^*) \qquad \text{(VI-8b)}$$

i.e., we have achieved the lower bound of L_2. See also [30].

More surprising is the fact that γ_2^* can generally be chosen to be affine in u_1, i.e., $u_2 = au_1 + b$ where a and b are constants to be determined by the two equations (VI-8a) and (VI-8b). Since (VI-8a) is satisfied by choice, we can generally determine a and b so long as γ_1^0 in (VI-8b) possesses the property that either a or b can be solved for in terms of the other and u_1^t, a most reasonable requirement. Note that the above illustration can be extended to the stochastic case and is not dependent on any LQG properties. What we have here is the ability of one DM to completely influence the decision of another because of the dynamic nature of the information structure.

This idea of incorporating one DM's action in another DM's strategy as a generalized incentive or threat can be traced back to the literature on repeated games [28]. For example, in zero sum repeated games, one player may be forced to disregard some useful information in making his decision lest his actions will reveal this information to his opponent. Threat of retribution may also create desirable outcomes in repeated games, such as Prisoner's dilemma, where honest cooperation free from cheating is required.

VII. Concluding Remarks

In a decision and control context, the ultimate purpose of acquiring information is to make better decisions. This naturally raises the question of the value of information. For example, while the knowledge of the winning lottery numbers in last week's and next week's drawing have the same information content in the Shannon sense, they have very different values. The study of information structure eventually should lead to a cost-benefit analysis of "who should know what?" However, in general we are far from being able to answer such a query systematically and in a unified manner. Several difficulties are involved. First of all, our understanding of the subject of dynamic information structure is meager. One purpose of this paper is to convey a sense of both the difficulty and the richness of the subject. Much more insight, for example, on the matter of "signaling" and "incentive structure" are needed before more sophisticated application of team theory can be realized. In this sense, much of the current work can be thought of as preparatory and basic. Secondly, the question of value of information implies the ability to determine the optimal solution to decision problems with or without the particular piece of information in the information structure. In view of the difficulties of solving decision problems in dynamic information structure, one tries to device methods to order information structures without having to solve the decision problems involved. For example, if the information structure η' is a garbled version of another structure η then η is more valuable than η' for all payoff functions which are not functions of the garbling noise.[12]

Finally, the problem of information structure design is related to the question of organization theory. It is often conjectured that organizations are formed due to the requirement of informational efficiency. However intuitively appealling such a conjecture may be, to formalize and make precise such statements require results and concepts yet to be developed.

It is likely that the problems in information structure will remain with us for years to come. Many interesting confluences of diverse disciplines are taking place around this subject. We have already touched on economics, decision and control, and game theory. A fourth possibility in the future lies with computer science and distributed data base systems. Although current efforts in this subject are directed along rather different lines, we mention one example of such possibilities [24], [25].

Note: The optimal decision rules are:

for B	go if it rains
	go if it shines
for H	same as B.

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Spectral Analysis and Adaptive Array Superresolution Techniques

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Abstract—Recent nonlinear "superresolution" techniques reported in the field of spectral analysis are of great interest in other fields as well, including radio-frequency (RF) adaptive array antenna systems. This paper is primarily a "cross-fertilization" treatise which takes the two most popular nonlinear techniques, the Burg maximum entropy method and the maximum likelihood method, and relates them to their similar nonlinear adaptive array antenna counterparts, which consist of the generic sidelobe canceller and directional gain constraint techniques. The comparison analysis permits an examination of their principles of operation from the antenna spatial pattern viewpoint, and helps to qualify their actual superresolution performance.

A summary of the resolution performance of several adaptive algorithms against multiple-incoherent sources is provided, including a universal graph of signal-to-noise ratio (SNR) versus source separation in beamwidths for the case of two equal-strength sources. Also, a significant dividend in the easy resolution of unequal-strength sources is reported. The superresolution of coherent spatial sources or radar targets is more difficult for these techniques, but successful results have been obtained whenever sufficient relative motion or "Doppler cycles" are available. Two alternate adaptive spatial spectrum estimators are suggested, consisting of a circular array predicting to its center point, and a new "thermal noise" algorithm.

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This contribution is derived largely from [1], and also, a subsequent report [2].
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I. Introduction

NONLINEAR spectral analysis techniques are currently a subject area of intense interest in the fields of spectrum analysis, geophysics, underwater acoustics, and radio-frequency (RF) array antennas. The major reason for the